Math Pathways: Defining Problems and Challenges in the State of Colorado

Working Paper

Colorado Math Pathways Task Force
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COLORADO MATHEMATICS PATHWAYS

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Introduction

Mathematics pathways refer to the course or the sequence of courses that students take to meet credit or program requirements and to prepare them to use mathematics for their careers, for making sense of the world, and for their own sense of enjoyment and empowerment. Typically, discussions of mathematics pathways focus on the courses students take as they progress through secondary (high school) and postsecondary education (college or other career training) and how the availability of multiple pathway options allows students to take advantage of opportunities best suited to their unique needs. Ideally, each student is in a mathematics pathway that:

- Aligns with their aspirations,
- Supports them with the resources needed to succeed,
- Has equitable access to and success in the courses, and
- Provides a smooth transition from secondary education to postsecondary opportunities.

The Central Problems to Solve With Improved Math Pathways

For most students, the sequence of math courses they experience in secondary and postsecondary education is determined by some judgment of their mathematical ability. For decades, it has been the policy and practice of schools to place students in math classes that vary by some perception of rigor. Students get placed in "fast" tracks (often called honors, accelerated, or higher-level) or "slow" tracks (sometimes called basic, fundamental, or lower-level), or some track or level in between. These placement decisions can be made as early as 5th or 6th grade, and once students are placed on a lower-level track, they are unlikely to ever move to a higher-level track. Many schools that practice this kind of "tracking" do so in the belief that matching students' abilities to the level of content and instruction is in their students' best interest., However, research on the effects of tracking is mixed, at best, with some studies showing academic benefits for at least some students and other studies showing harm to most students. The major mathematics education organizations, including the National Council of Teachers of Mathematics (NCTM), NCSM: Leadership in Mathematics Education, and the Association of State Supervisors of Mathematics (ASSM) have all published statements against ability-based tracking.

Regardless of the track, the typical high school mathematics pathway has been designed to take students to one place: Calculus. The country's focus on calculus in the mathematics curriculum can be traced back to Sputnik and the space race of the 1960s when it was a national priority to apply mathematics, physics, and other sciences to solve space-age problems. Over decades,

there has been a dramatic rise in the number of students taking the AP Calculus exams and in calculus course enrollments more generally. For students needing calculus in their future careers, the greater availability of calculus courses has been a hard-earned, multi-decade achievement, although there is still much work to do to ensure these opportunities exist in all schools for all students.

For the estimated 80 to 90 percent of students who will pursue careers, college majors, or other options that don't use calculus, being placed in a calculus pathway might represent a *missed opportunity*. We still live in a space age and we still need some students to learn calculus. But we also live in an information age, and many students will have career interests that require opportunities to learn advanced mathematics and statistics skills that will help them collect, manage, and make sense of data. There is another group of students whose futures are unlikely to require skills in either calculus or data analysis, but for whom a strong, broad foundation in modern, applicable mathematics focused on quantitative literacy and reasoning would be more beneficial than pursuing advanced topics required in calculus or statistics.

These two central problems – tracking students by ability, and being overly calculus-focused in the tracks that do exist – are what improved mathematics pathways hope to overcome. We can move towards our ideals stated above, where students are on paths aligned with their interests and getting the support they need. These supports would help students transition smoothly from high school to college or whatever else may come next. This will require collaboration and fresh thinking across our educational systems, but the momentum to make these changes is already here and people are ready to keep moving forward.

A Decade of Progress

Colorado has made substantial progress with its mathematics pathways over the past decade, especially in higher education. Colorado's first math pathways task force met from 2014 to 2016 to focus on "gateway" college mathematics courses, which are the entry-level courses that carry college credit. The prevailing practice at the time had been to rely on College Algebra as the default gateway course, regardless of students' aspirations, and to enroll students in one or more "developmental" non-credit-bearing courses for students who did not demonstrate readiness for the gateway course according to state-determined measures of mathematics proficiency.

In the years since Colorado's first math pathways task force, our higher education institutions have largely shifted away from enrolling students in developmental courses or defaulting to College Algebra as the gateway course. Developmental education (DE) courses are "prerequisite" courses, in which students take perhaps one, two, or three semesters of DE *before* working their way into the gateway course that satisfies the requirements for their

program or major. Instead of relying on DE, it has become more common to enroll students into co-requisite math courses (sometimes called supplemental academic instruction, or SAI) that support student success *while* they are enrolled in their gateway course. Implementation of the previous task force's recommendations varies somewhat, but the results of this shift away from DE and towards SAI have been overwhelmingly positive. The Colorado Community College System has decreased its student enrollment in developmental courses from 14.9% in 2016-2017 to 4.0% in 2020-2021. In the 2020-2021 school year, the math gateway course completion rates were: 28.2% for students who had taken only a DE course previously, 55.6% for students who had taken both a DE course previously as well as a SAI course concurrently, and 63.3% for students who took only an SAI course concurrently (no DE course previously). As to what the required gateway course(s) might be, it can vary from institution to institution and program to program, but at the Community College of Denver, College Algebra is the most commonly required math course. After College Algebra, the next most popular gateway courses are Introduction to Statistics, followed by Mathematics for the Liberal Arts. Many programs do not require a specific math course, allowing students to choose any Guaranteed Transfer mathematics course from a list of several options.

Colorado's first math pathways task force did not attempt to directly impact K-12 education, but publications and conversations at a national level have begun to make impacts in Colorado's high schools. NCTM's publication of *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* in 2018 has prompted conversations about the structure and content of high school courses and how to extend the pathways work from higher education into K-12.

The Challenge of Change

Making the kinds of changes in Colorado's high schools that we have already seen in our colleges and universities will not come without significant challenges. Mathematics education is deeply engrained in our educational systems, with inertia built up over decades that will be resistant to change. Our chances of success increase as we think clearly about what these challenges are and how best to overcome each one, without accidentally ignoring others. In studying these issues for the year, the Colorado Math Pathways Task Force has arranged these challenges into three categories: *policy challenges*, *practice challenges, and perception challenges*.

Policy Challenges

Policy challenges refer to those things affecting math pathways that can be found in state statutes or requirements enforced by the Colorado Department of Education (CDE), the Colorado Department of Higher Education (CDHE), and the Colorado Community College System (CCCS). Policy challenges also refer to decisions made and enforced by boards of education

(state and local), testing companies like The College Board, or other large-scale, influential policy-making bodies that have a widespread influence over how mathematics pathways are designed in Colorado.

Some policy challenges exist because of contrasts between Colorado's higher education and K-12 educational systems. For example, in the Colorado Community College System, all institutions share common course titles and course objectives for their gateway math courses. This level of consistency supports guaranteed transfer agreements between institutions and a wider understanding of those courses and their objectives across the higher education system. More variability is found in Colorado's four-year institutions, where it is less common to find advisors and faculty who share such common knowledge of their courses. Still, this contrasts sharply with Colorado's high schools, where the state's tradition of local control leaves course and credit decisions – including the number and kind of courses required for graduation – completely up to local, independent school boards. As a result, high schools across the state use a variety of course titles and descriptions for their math courses, and there is great variability in how schools arrange their mathematics courses into pathways that lead to graduation and college admission. The Colorado Department of Education collects data about courses students take in the state's high schools, but because the data is infrequently used or publicized, often minimal effort is made by schools to ensure their course descriptions are accurate and represent some kind of consensus with other schools in the state. The difference in Colorado's high school and higher education policy and governance structures is itself not a problem–at least not fundamentally–but it does present a challenge when the K-12 and higher education systems do not work to align themselves with one another. For any set of mathematics pathway recommendations to see widespread adoption, it will be necessary to have a savvy understanding of these governance structures and to know where and how to gain support for new ideas.

As another example of the contrast between Colorado's higher education and K-12 systems, state statute requires the Colorado Department of Education to enforce a set of "graduation guidelines." Unlike other states requiring two, three, or four math credits to graduate, Colorado has a "menu of options" from which schools can select one or more ways to have their students show mathematical proficiency. Options on this menu include earning above a minimum standardized test score or completing a college-level course or credential, but there is no option to simply accumulate course credits. Meanwhile, the Colorado Department of Higher Education maintains HEAR, the Higher Education Admissions Recommendations, which suggests that universities require four credits in mathematics for admission, including two credits in algebra and a credit in geometry. Both the menu of options and HEAR attempt to describe the mathematical proficiencies expected of students at a specific point in their lives, but the two systems take very different approaches.

Some examples are better characterized as misalignment than just a difference in approaches. In 2015, the Colorado Legislature re-wrote existing academic placement rules for post-secondary students which said that multiple measures (for example, high school Grade Point Average and a SAT score) must be used to place students into gateway math and English courses. Instead of the previous practice of relying on a single test score, having more data from more measures should help advisors and faculty make informed choices when choosing which gateway course is right for a student, in addition to any corequisite courses or other support the student may need. This legislation, however, did not directly affect high schools. Because high schools tend to focus on providing a pathway to calculus, the courses and assessments students take assume calculus is the goal. As a result, an incoming undergraduate will likely have multiple measures designed to measure progress along a calculus pathway, like an SAT score or an Algebra 2 grade, but not measures designed for non-calculus pathways.

The Colorado Academic Standards are the single policy instrument that exerts the greatest influence over what is taught and learned in preschool through Grade 12. In mathematics, Colorado's standards closely follow the Common Core State Standards, which contain well-designed progressions for learning mathematics in preschool through Grade 8. For high school, however, the standards are not individually assigned to Grades 9, 10, 11, and 12. Instead, the standards exist as a single set of high school standards that are not broken out by grade or course. Because the progression for learning across grades is unclear in the standards, it adds to the challenge of creating high school mathematics courses and pathways. In addition, Colorado's mathematics standards are most supportive of students pursuing a pathway to calculus but are less supportive of students pursuing non-calculus pathways, especially pathways supporting the liberal arts or career and vocational tracks. Although state statute calls for a "seamless" system of academic standards that also incorporates the goals of career and technical education (CTE) and higher education, the current standards do not make this explicit. In addition, schools can graduate students with, what is considered a lesser degree, with Algebra and Geometry content. What qualifies as Algebra and Geometry content is not defined or measurable. This leaves high school educators and administrators in Colorado with simple questions without clear answers, such as "What math should be in a financial algebra course?" or "What distinguishes a statistics course from a data science course?" Colorado's mathematics standards are scheduled for review and revision in 2026, but until more structure can be defined to the mathematics content to be taught to high school students, building well-defined, cohesive high school math pathways will remain a challenge.

The broader issue of policy challenges in the transition from secondary to postsecondary education is currently being taken up by Colorado's Secondary, Postsecondary, and Work-Based Learning Integration Task Force. This task force, prompted by the passage of House Bill 22-1215, is examining the policies and systems in place that encourage and prepare students for college and the workforce. The Secondary, Postsecondary, and Work-Based Learning Integration Task Force, whose work is still in progress, has called for what they describe as

"the big blur," where obstacles that divide the secondary and postsecondary education levels are removed and students experience a more continuous progression of education and skill development as they enter adulthood. That Task Force has also identified a lack of longitudinal data in Colorado, meaning we currently do not have the ability to collect data and follow students from high school to postsecondary and determine how different programs or decisions have improved their outcomes. Colorado has numerous programs, including several established in legislation, designed to foster students to a productive life after high school. But each of those programs comes with different requirements, budgets, applications, and levels of oversight. The details of all these programs are not problems for the Colorado Math Pathways Task Force to solve; however, given the effect these programs have on students seeking math courses that help them transition out of high school and into college or the workforce, the two task forces will need to work together to find solutions that serve students of mathematics as well as all students more generally.

Practice Challenges

Practice challenges refer to the habits, traditions, and local decisions determining how mathematics pathways are made available to students. This may include the courses that schools make available to students, the topics included in those courses (when those things are not a matter of policy), and how students are placed into those courses.

To understand practice challenges, it is probably best to start by taking the perspective of a student. Although much of the effort to reform mathematics pathways focuses on improving options for high school students, a typical student entering high school may find that their path is at least partially already determined. Because tracking students by ability is a common practice, and because many schools start placing students into ability-based tracks in middle school, many students will either find themselves enrolled in a course on a pathway designed for college-bound students (which probably leads to a calculus course) or, if they were not in an "advanced" or "accelerated" track in middle school, they may start high school feeling like they are already behind their peers. Given what we've identified as the two key problems of mathematics pathways – they rely on perceptions of student ability instead of student interest, as well as being overly focused on calculus – you can see how the practices and traditions middle schools and high schools rely on to place students is perhaps the most direct challenge to reform. The importance of this challenge is elevated further by the biased and racist ways students are assigned to pathways, as evidenced by the disproportionate number of Black, Latinx, low-socioeconomic status, and disabled students assigned to "lower" pathways.

Beyond the initial placement of students in a high school pathway, we are challenged by the lack of support for high school students who fail one or more mathematics courses. In practice, schools design their mathematics pathways with positive assumptions about student success.

However, we know that significant numbers of students fail at least one semester of a mathematics course. Schools tend to have a limited number of ways of helping those students, such as having students retake the course, enroll in summer school, or complete some form of online credit recovery. When schools build their mathematics pathways without considering the support that will be needed to help students who have stumbled, too often the result is that those students "fall off" a pathway that might prepare them for college or a career and are placed in a situation where they are merely trying to accumulate credits so that they may graduate. Getting those students to graduation can be seen as a success, but this success is diminished if the students find themselves mathematically unprepared for where they wish to be headed next.

Both of the above challenges raise questions about the capacity that teachers and schools have to provide improved mathematics pathways to students. Even for a school that wants to provide a variety of interest-based pathway options along with adequate support for students who struggle, the school may not have enough teachers on their staff or time in their schedules to meet all students' needs. The logistical challenges here are significant, especially in Colorado's smaller schools where there may only be one or two high school mathematics teachers in a school. While large class sizes can be a challenge in large schools, small schools often have the opposite problem. When class sizes are small, revenue coming to the school tends to be small, resulting in fewer teachers teaching more courses at once to meet the needs of relatively few students. Not only is it a challenge to stretch a teaching staff thinly over many courses, these teachers often receive smaller salaries than their counterparts in larger schools. Ideally, high schools will have pathways that align with the calculus, statistics, and quantitative literacy options found in many of Colorado's colleges and universities, but offering all three will likely require collective problem solving and resource sharing beyond what schools are currently accustomed to.

Even when enough teachers are available to offer a wider variety of course options, not all teachers have the knowledge, training, and materials to teach those courses well. Often, training and materials exist, but they aren't widely available or well-known. Some of this can be influenced by policy, as teacher endorsement standards for mathematics teachers require a variety of coursework. Still, many teachers feel unprepared to teach courses like statistics and data science because their training was limited and there was never a tradition or expectation that they would have many opportunities to apply that training in their teaching careers. The situation in preservice teachers' experiences is mirrored in inservice teachers' experiences, where opportunities and materials may exist but the traditions and expectations do not. As such, it is a practice challenge to rethink our traditions and expectations so that the preparation teachers receive and the materials they are familiar with better reflect student interests and the course options available to students with improved mathematics pathways.

Perception Challenges

Perception challenges refer to the beliefs and feelings that people have about what math should be taught and learned, how it should be taught and learned, and who deserves or needs access to that math. This includes perceptions held by educators, policymakers, parents, students, and anyone else whose beliefs and feelings affect math pathways.

Most people, adults and children alike, have feelings about mathematics and their mathematics learning experiences. Unfortunately, many of those feelings are negative. People readily claim they "aren't a math person" or that they "hate math." Yet, paradoxically, some of these people reject the idea that the content and teaching of mathematics should change, and resist the idea that it can and should be improved. For some, math course placement is a simple marker that determines who is "smart" or "bright" from who is not, or who has "potential." This has a direct impact on the improvement of math pathways because we want students in pathways aligned to their interests, not their perceived ability. A high school senior interested in business, sociology, or journalism should not be told by a guidance counselor to take calculus instead of statistics because "you need calculus to impress college admissions officers." That student may wish to take calculus and may find some benefit from it, but taking calculus simply to mark oneself as "smart" is not a sufficient reason to take a course. Reforms to mathematics pathways will have to challenge peoples' perceptions about where "smartness" lies in math as we move towards a system in which a range of mathematical abilities can be found within each pathway, and non-calculus pathways aren't seen as lesser options for lesser students.

To sharpen the point about the perception challenge that calculus equates with "smartness," we need to carefully think about the perceptions of students and parents, especially those from populations historically underrepresented in mathematics and STEM fields. For students and parents who may identify as Black, Latinx, disabled, financially insecure, or who did not learn English as their first language, course and pathway options have often been used as a way to segregate them from a dominant population and as a way to provide them with fewer or lower-quality resources. Therefore, anyone working to reform mathematics pathways should expect and welcome skepticism from these parents and students. On the flip side of this problem, the perceptions of students and parents who feel they've benefited from this kind of segregation will also need to be addressed. For some parents, typically the White and the affluent, choosing their child's mathematics pathway has as much to do with choosing their child's peer group as it does learning mathematics and preparing for a future career or college major. Addressing these perceptions will require strong policies and practices in addition to naming and confronting historical inequities and the perceptions that now exist as a result.

In addition to these perceptions, successful reform of mathematical pathways must address people's fear that change will put students at risk. Local control of school governance gives

Colorado's K-12 schools the freedom to be innovative, but that sense of freedom is counterbalanced by the fear that being innovative with mathematics courses and pathways will put students at risk of getting a lower SAT score, not graduating, or not being accepted into college. For decades, the "safe" pathway has been a four-year sequence of Algebra 1, Geometry, Algebra 2, and Precalculus, with an AP or college calculus option for students who complete Algebra 1 in 8th grade or otherwise get a year ahead. Even offering courses such as Integrated Math 1, Integrated Math 2, and Integrated Math 3 is disliked by some and dismissed as too risky, even though these courses have existed since the 1990s and are the required option in some states. For a bolder set of changes, such as creating pathways designed to prepare students for statistics and quantitative literacy, teachers and school leaders will need confidence that they aren't putting students at risk, and they will need the support of parents who feel the same way. A big part of meeting this challenge will likely require getting multiple districts to undertake similar reforms together so that no one school or district feels like they are making local decisions that are unrecognizable or contrary to the decisions of everyone else.

Conclusion

Math pathways play a critical role in shaping students' mathematical identity and skillset as they prepare for higher education, the workforce, and life in a modern world. However, our current pathways, which track students by perceived ability and over-focus on calculus, need improving. To see improvement, we have policy challenges, practice challenges, and perception challenges to overcome. By working together and learning from successful experiences, we can create math pathways that provide equitable access to high-quality and relevant mathematics education, ensuring students' success in the modern world.